Cosmic ray ensembles as signatures of ultra-high energy photons interacting with the solar magnetic field

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Propagation of ultra-high energy photons in the solar magnetosphere gives rise to cascades comprising thousands of photons. We study the cascade development using Monte Carlo simulations and find that the photons in the cascades are spatially extended over hundreds of kilometers as they arrive at the top of the Earth's atmosphere. We compare results from simulations which use two models of the solar magnetic field, and show that although signatures of such cascades are different for the models used, for practical detection purpose in the ground-based detectors, they are similar.

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Introduction: Detection of ultra-high energy (UHE) photons will have a significant impact on the understanding of fundamental science. As an example, dark matter (DM) searches up to the electroweak scale (~ 100 GeV) so far have not been able to produce conclusive evidence of DM particles [1–3]. For this reason, it is quite instinctive that the mass regime corresponding to the other natural scales– the GUT and the Planck scale be explored for the DM search [4, 5]. A common method preferred by many for DM search has been the indirect search, which essentially hinges on the detection of products of DM particle decays and annihilation. Various proposed models of particle interactions predict that products of such interactions consist of UHE photons and standard model (SM) particles with a possibility of other elementary particles which do not fit into the SM [6]. Detection of UHE photons will also help substantiate the Greisen–Zatsepin–Kuzmin (GZK) effect, a steepening of cosmic ray energy spectrum around 4×10^{19} eV as a consequence of interaction of UHE cosmic rays (UHECRs) with cosmic microwave background radiation [7, 8].

Widely used techniques of UHE photon detection rely on two main approaches, first, analyses based on parameters (e.g., $X_{\max}$) from the reconstructed longitudinal profiles of development of extensive air showers (EASs) initiated by UHE photons [9], and the other based on observables derived from signal recorded by ground-based detector arrays from the secondary particles of EASs [10]. In principle, both approaches should be able to distinguish between photon- and hadron-initiated showers. The photon-initiated showers are expected to have deeper $X_{\max}$ compared to the hadron-initiated ones, and the particle contents for the two types of showers are expected to be different– hadronic showers being more muon-rich than the other. The most up-to-date results from searches implementing these techniques have reported not only the non-observation of (significant) photon candidates in UHECR data, thus enabling us to place stringent upper limits on UHE photon fraction (flux) [9, 10], but also observation of an excess of muons in data compared to what one would expect from simulations of hadronic showers [11, 12]. Given such a discrepancy between the measurements and the results from simulations of hadronic showers, which is possibly due to the lack of complete understanding of the physics at the UHE regime, it is very appealing to revisit the UHE photon scenario also with a different approach.

The alternative approach presented in this Letter is based on the electromagnetic cascading of UHE photons traversing regions nearby the Sun. Simulation results from a study of such a cascading process were presented in [13], which give an expected size of a footprint of core part of the cascade at the top of the Earth's atmosphere. The footprint is expected to be a highly prolate ellipse with a size of the order of a few kilometers. In our simulations, we take into account the more accurate physics of cascade development and tracking of the cascade particles so that we are able to characterize the particle distribution better. The cascading process starts when a UHE photon experiences solar magnetic field component transverse to the direction of its trajectory sufficiently large for magnetic pair production. The electron-positron pairs thus produced undergo a magnetic bremsstrahlung process and emit photons as they propagate in the magnetic field. Also, among the emitted photons, those with sufficiently high energy will undergo magnetic pair production and repeat the process. As a consequence, a cascade comprising several thousand photons and several e^+e^- develops in the region nearby the Sun. Although deflections suffered by the e^+e^- during their propagation are very small when considered only within these regions, they give rise to an extended spatial distribution of cascade particles after propagating through the Sun-Earth distance ($\sim 1.5 \times 10^{11}$ m). For UHE photons heading towards the Earth through the regions in the Sun's vicinity, a unique particle distribution is expected as the cascade reaches the Earth. Such a cascading of UHE photons can occur even in the presence of the geomagnetic field [14]. However, cascades produced in the geomagnetic field, which are called *preshowers*, comprise only few hundred particles and have very narrow spatial distribution (< 1 m). Due to this fact, they are practically indistinguishable from the cascades without the preshower effect unless they originate at much higher altitude or arrive at the Earth's atmosphere at near horizontal direction. In the following part of this Letter, we refer to the Sun-initiated cascades as *super-preshowers* (SPSs) in light of similar development mechanism as that of preshowers but with much larger number of secondary particles.

Simulation: The treatment of most of the physics processes involved in the simulation of SPS development has been adopted from the PRESHOWER program [14]. We have used the formalism for magnetic pair production from reference [15]. For $n_{\rm photons}$ UHE photons propagating through a magnetic field (H), the actual number of e^+e^- pairs produced ($n_{\rm pairs}$) is given by,

$$n_{\rm pairs} = n_{\rm photons}\{1 - \exp\left[-\alpha(\chi)\,{\rm d}l\right]\}, \qquad (1)$$

where dl is the photon path length and $\alpha(\chi)$ is the photon attenuation coefficient, a function of parameter $\chi \equiv \frac{1}{2}\frac{h\nu}{m_{\rm e}c^2}\frac{H}{H_{\rm cr}}$, $H_{\rm cr} \equiv \frac{m_{\rm e}^2 c^3}{e\hbar} = 4.414 \times 10^{13}$ G being the natural quantum-mechanical measure of magnetic field strength. In an ultrarelativistic limit, if $H \ll H_{\rm cr}$, $\alpha(\chi)$ can be expressed as

$$\alpha(\chi) = \frac{1}{2}\frac{\alpha_{\rm em}}{\lambda\!\!\!^{-}_{\rm c}}\frac{H}{H_{\rm cr}}T(\chi), \qquad (2)$$

where $\lambda\!\!\!^{-}_{\rm c}$ is the Compton wavelength of the electron and $T(\chi)$ is a dimensionless auxiliary function which can be approximated by

$$T(\chi) \simeq \frac{0.16}{\chi}K_{1/3}^2\left(\frac{2}{3\chi}\right), \qquad (3)$$

where $K_{1/3}$ is a modified Bessel function. Provided the path length under consideration (dl) is fairly small, Eq. (1) can be expressed as a probability of conversion of UHE photon into e^+e^- pair ($p_{\rm conv}$) within the interval dl. Thus, we have

$$p_{\rm conv} = 1 - \exp\left(-\alpha(\chi)\,{\rm d}l\right) \simeq \alpha(\chi)\,{\rm d}l, \qquad (4)$$

which for a much larger distance L takes the form,

$$P_{\rm conv} = 1 - \exp[-\int_0^L \alpha(\chi)\,{\rm d}l]. \qquad (5)$$

The probability of conversion of a UHE photon into e^+e^- pair is evaluated using Eq. (4). Also, a fraction of energy carried by a pair-member (ε) is chosen from the distribution

$$\frac{{\rm d}n}{{\rm d}\varepsilon} \approx \frac{\alpha_{\rm em}H}{\lambda\!\!\!^{-}_{\rm c}}\frac{\sqrt{3}}{9\pi\chi}\frac{[2+\varepsilon(1-\varepsilon)]}{\varepsilon(1-\varepsilon)}K_{\frac{2}{3}}\left[\frac{1}{3\chi\varepsilon(1-\varepsilon)}\right] \qquad (6)$$

following [16].

As the conversion probability of UHE photons to e^+e^- pairs, their trajectories and characteristics of magnetic bremsstrahlung radiation emitted thereof depend on the magnetic field experienced by these particles along their trajectories, it is important that we incorporate a realistic solar magnetic field model in our simulation. Unsurprisingly, owing to the dynamic nature and complexity of the Sun's magnetic field, a model that can characterize the magnetic field completely is far from being achievable. Thus, we proceed first with a simple dipole model of solar magnetic field and later with another analytical model called the dipole-quadrupole-current-sheet (DQCS) model [17]. For the dipole model, the magnetic moment of the dipole producing the field used in the simulation is 6.87×10^{32} G $\cdot$ cm^3. Although this is not a very realistic model, using it for the solar magnetic field in our simulations allows us to study the effects of orientation of considered dipole on the expected distribution of the particles in the SPS as they arrive at the top of the Earth's

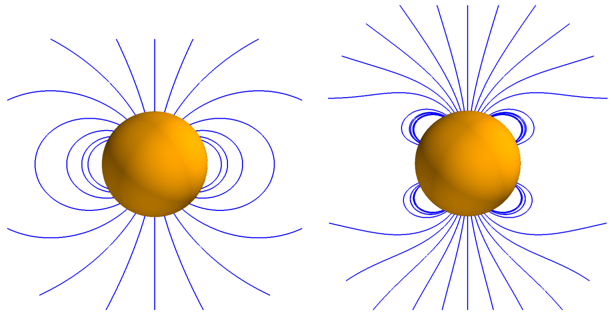

FIG. 1. Magnetic field models used in the simulation

atmosphere. This will give us an idea of how the SPS development is affected by an evolution of solar magnetic field for example over a solar cycle. In addition, it serves for a comparison to the results obtained from the other model. The DQCS model on the other hand, is more realistic and gives a more reasonable magnetic field even in the interplanetary regions. Inclusion of this model in the simulation thus provides a more accurate tracking of e^+e^- on their way towards the Earth, and better treatment of magnetic bremsstrahlung process.

We have introduced time and space tracking for particles in the cascade so that we get their arrival time distribution and lateral distribution as they arrive at the top of the Earth's atmosphere. Given a particle with kinetic energy E and charge q, propagating along the direction $\hat{\boldsymbol{v}}$ in a region defined by a magnetic field $\boldsymbol{B}$, the equation describing its motion as a function of time t can be written as,

$$\frac{\mathrm{d}\hat{\boldsymbol{v}}\,(t)}{\mathrm{d}t} = \frac{qc^2}{E}\hat{\boldsymbol{v}} \times \boldsymbol{B} \quad . \tag{7}$$

The direction of propagation of a particle after it traverses a distance Δs in time interval Δt can be approximated by using a Taylor series expansion of $\hat{\boldsymbol{v}}\,(t+\Delta t)$ around t,

$$\hat{\boldsymbol{v}}\,(t+\Delta t) \approx \hat{\boldsymbol{v}}\,(t) + \frac{\mathrm{d}\hat{\boldsymbol{v}}\,(t)}{\mathrm{d}t}\Delta t,$$

which takes the form

$$\hat{\boldsymbol{v}}\,(t+\Delta t) \approx \hat{\boldsymbol{v}}\,(t) + \frac{qc^2}{E}\left(\hat{\boldsymbol{v}} \times \boldsymbol{B}\right)\Delta t, \tag{8}$$

after substituting $\frac{\mathrm{d}\hat{\boldsymbol{v}}\,(t)}{\mathrm{d}t}$ from Eq. (7). We implement such a particle motion by choosing an appropriate Δs which is split into two halves each equal to $\Delta s/2$. In the first half of the time interval $\Delta t/2 = \Delta s/2c$, the particle is propagated with the current direction vector which is then updated using Eq. (8) and is propagated with the new direction vector for the latter half of the interval.

Using an expression for the spectral distribution of energy radiated by ultra-relativistic electron from [18]

$$f(y) = \frac{9\sqrt{3}}{8\pi}\frac{y}{(1+\xi y)^3}\left\{\int_y^\infty K_{\frac{5}{3}}(z)\,\mathrm{d}z + \frac{(\xi y)^2}{1+\xi y}K_{\frac{2}{3}}(y)\right\}$$

where parameter $\xi = \frac{3}{2}\frac{H_\perp}{H_{\mathrm{cr}}}\frac{E}{m_{\mathrm{e}}c^2}$, E and m_{e} are energy and rest mass of electron respectively and y is a function of emitted photon energy $h\nu$ defined by

$$y\,(h\nu) = \frac{h\nu}{\xi\,(E - h\nu)},$$

one can obtain the probability of emission of a bremsstrahlung photon from a sufficiently small path length $\mathrm{d}l$. As has been derived in [14], the probability can be written as

$$P_{\mathrm{brem}}\,(B_\perp, E, h\nu, \mathrm{d}l) = \mathrm{d}l\int_0^E I\,(B_\perp, E, h\nu)\,\frac{\mathrm{d}\,(h\nu)}{h\nu}, \tag{9}$$

with

$$I\,(B_\perp, E, h\nu) \equiv \frac{h\nu\,\mathrm{d}N}{\mathrm{d}\,(h\nu)\,\mathrm{d}l},$$

where $\mathrm{d}N$ is the number of photons with energy between $h\nu$ and $h\nu + \mathrm{d}\,(h\nu)$ emitted over the path length $\mathrm{d}l$.

In addition, we have included the angular distribution of emitted synchrotron photons in our simulations. Since electrons are ultra-relativistic, we take the half-opening angle of emitted synchrotron photons to be equal to $1/\gamma$, γ being the Lorentz factor of the electron. The azimuthal angle of emitted photon is randomly chosen from a uniform distribution $U(0, 2\pi)$ [19].

Results and Summary: We performed simulations for various representative cases of primary UHE photons traversing the Sun's vicinity on their way towards the Earth. The solar magnetic field component transverse to the propagation direction of primary UHE photon has sufficiently large strength for pair production only in a small fraction of the path length close to the Sun. Emission of synchrotron photons from the e^+e^- pair thus produced also occurs mostly in the region close to the Sun. Thus, almost the entire cascade development occurs in the close vicinity of the Sun.

The electron and the positron, although travelling along slightly different tracks, experience practically the same transverse magnetic field strength. The electron and the positron are deviated in opposite directions approximately in the same plane, when considered only in the small region where most of the cascade develops. The argument that their motion is approximately in the same plane comes from the fact that for the highly energetic e^+e^- travelling in a magnetic field of which the strength typically is much less than a Gauss, the gyroradius of the motion is much larger than the length of the

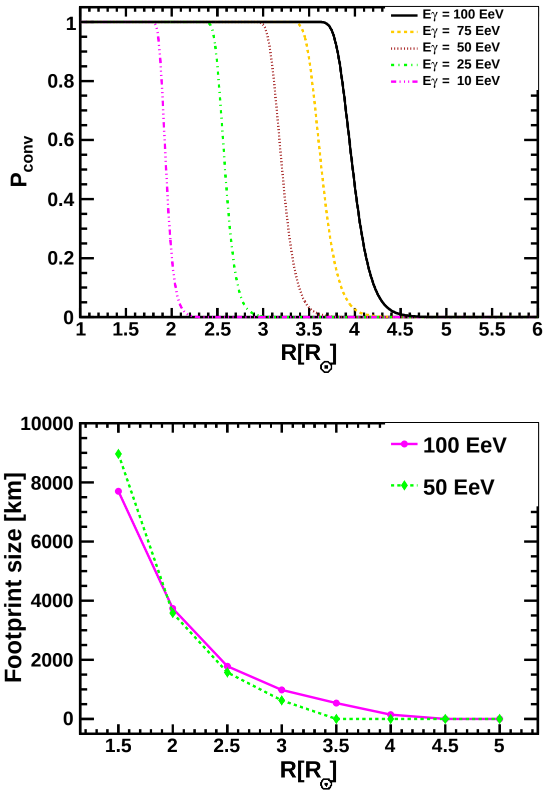

FIG. 2. Top panel: Probability of magnetic pair production ($\gamma \rightarrow e^+e^-$) as a function of the impact parameter for UHE photons heading towards the Earth form the Sun's vicinity. Bottom panel: Size of SPS footprint at the Earth as a function of the impact parameter. In both plots, primary UHE photons travel through the Sun's polar region.

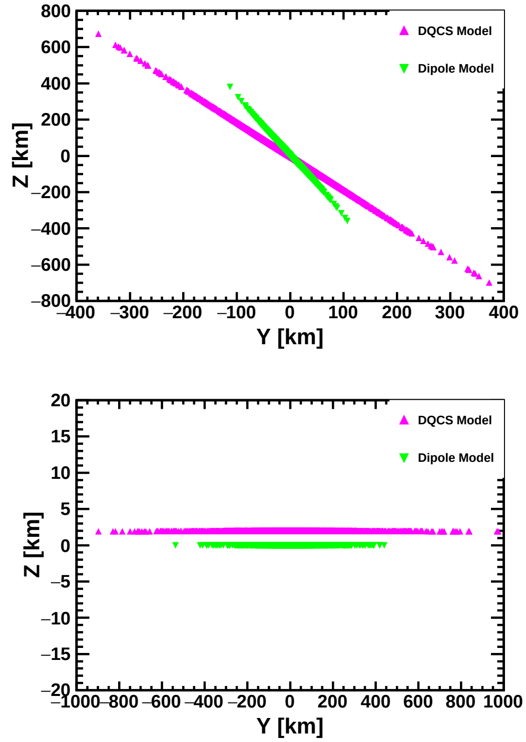

FIG. 3. Top panel: Spatial distribution of photons with energies $> 10^{12}$ eV arriving at the top of the atmosphere for an SPS produced by 100 EeV photon. The primary photon is directed towards the Earth such that the position of the closest approach has heliocentric latitude 45°. Results from DQCS (dipole) model are displayed. The impact parameter for the primary UHE photon is 3 $R_\odot$. Bottom panel: Same as the top panel but with the position of the closest approach having heliocentric latitude 90°. Also, distribution obtained using DQCS model is shifted by 2 km in positive y direction for the sake of clarity.

track where they experience this field. Synchrotron photons emitted from these ultra-relativistic electrons are extremely beamed in the forward direction of the latter, which gives rise to spatial distribution that has a very elongated footprint, when the cascade arrives at the top of the Earth's atmosphere. The probability of conversion of a 100 EeV UHE photon propagating towards the Earth from the Sun's vicinity as a function of its impact parameter is shown on the top panel of Fig. 2. The conversion probability is close to unity for impact distance as far as $4R_\odot$ from the Sun's center for 100 EeV photon, which translates to the fact that despite a small solid angle subtended by the Sun while viewed from the Earth, the effective solid angle relevant for SPS search is about 15 times larger at this energy. However, for lower energies around 10 EeV, the conversion probability is close to unity as far as $2R_\odot$, thus giving a region 3 times larger than the apparent size of the Sun viewed from the Earth. Also, on the bottom panel of Fig. 2, the footprint size at the Earth as a function of impact parameter is shown for SPSs produced by 50 and 100 EeV photons.

A salient feature of SPSs we observe in our simulation results is a very extended spatial distribution of cascade particles, apparently along a straight line, as the cascade reaches the top of the Earth's atmosphere. This extended footprint is a straightforward consequence of deviation of electron and positron along their tracks under the influence of (practically the same) solar magnetic field, and emission of extremely forward-beamed synchrotron photons from them as they propagate towards the Earth. Particle distribution results for two example cases are shown in Fig. 3. In both plots in the figure, $y = 0, z = 0$ corresponds to the point at the top of the Earth's atmosphere where the UHE photon would have landed, had there been no interaction on its way. Positive y and z axes point towards the East and the North directions respectively. Although the particle distribution is dependent on the solar magnetic field model used in the simulation as is evident in the figure, the nature of the particle distribution (i.e., a very extended spatial distribution) holds for both models. In the top panel of Fig. 4, particle distribution at the top of the Earth's atmosphere weighted by particle energy for an example simulation is shown.

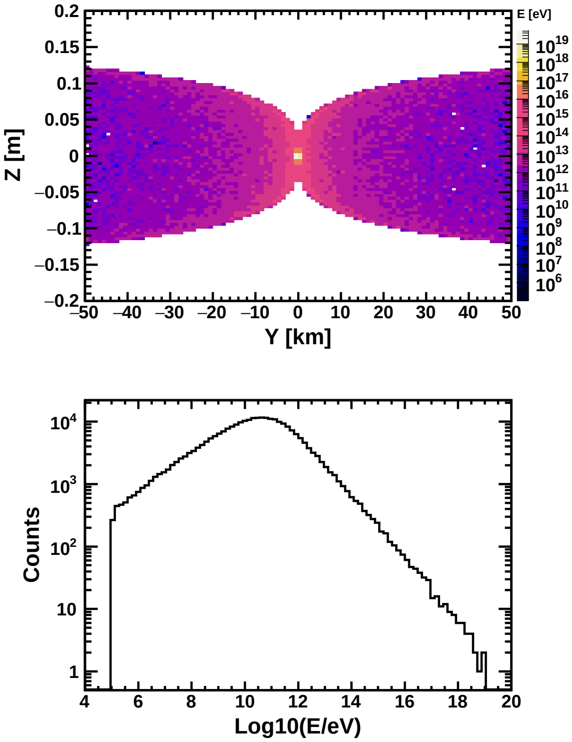

FIG. 4. Top panel: Distribution of energy of SPS photons arriving at the top of the atmosphere for an SPS produced by 100 EeV photon. The primary photon is directed towards the Earth such that the position of the closest approach has heliocentric latitude 0°, and its impact parameter is $3R_\odot$. Note the difference in the scales along x and y axes. Bottom panel: Energy distribution of SPS photons with energies larger than 10^5eV for the same SPS.

The central region of the cascade comprises the most energetic photons. Also, in the bottom panel in the same figure, energy distribution of photons in the SPS cascade is shown.

Detection of SPS cascades is limited by two major requirements. The first is the size of the detector itself, which should be big enough to detect SPS particles distributed over very large distances. The other obvious requirement is that the detector should be operational during the daytime. As such, only a large array of ground-based particle detectors like the Pierre Auger Observatory [20] and the Cosmic-Ray Extremely Distributed Observatory (CREDO), a global network of cosmic-ray detectors still under development [21] are suitable for SPS detection. Such detectors are capable of detecting secondary particles from the extensive air showers produced by SPSs.

As for the surface detector array of the Pierre Auger Observatory which has an effective area of about 3000 km^2, roughly 3000 cosmic-ray events above 10 EeV are detected annually. This translates to about 0.2 events per year from within the relevant region around the Sun. Thus, in the data collected in 15 years, $\sim$ 3 events are expected to arrive to the Earth from the relevant direction. Given the UHE photon limits we currently have [9, 10], the expected number of SPSs which completely lie within the Auger array during the Auger's lifetime is very small. Nevertheless, since we have large physics uncertainties at the UHE regime, the actual SPS footprint size could be considerably larger than what we obtain from our simulations allowing us to detect SPSs even if they do not land within the array entirely. SPS-like processes at other sites in the Universe as well as other physics processes might produce a "shower" of correlated particles, the cosmic ray ensembles (CREs), while they propagate in space. Thus, other stellar objects which have a magnetic field strength at least of the order of 0.1 G at their surfaces will also initiate SPS-like CREs. If we assume that a UHE photon undergoes an SPS-like process in regions of the Universe with relatively stronger magnetic field while heading towards the Earth and we have a cosmic ray detection framework that can detect two or more photons simultaneously at very distant locations, the "explorable horizon" for such process can be estimated using simple geometry considerations. From our SPS simulation results for 100 EeV photon, minimum distances between the most energetic ($>$ 1 EeV), and low energy (1 – 10 TeV) SPS photons as the cascade reaches the top of the Earth's atmosphere are both of the order of 10^{-7} m. Provided a framework which can detect these "close photons" in the CREs arriving as far as 10 000 km apart at the Earth from extragalactic regions or sites, the "horizon" is extended to $\sim$ 500 Mpc. For comparison, the mean free path for gamma-rays at 1 EeV (1 TeV) is of the order of 100 kpc (500 Mpc). Photon splitting in strong magnetic fields in the proximities of neutron stars [22, 23] is another process which is capable of producing CREs, of which the estimation of the expected signature at the Earth requires a dedicated study and will be performed in the near future. Although we are not certain about the expected rate of CREs, these, together with SPSs constitute a yet-unchecked scenario that is easy to verify and has a potential of opening a new window to the Universe.

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